Contest Problem Set 12306

Sprint Round Problem 5

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Problem

Suppose N is a positive integer such that $10N$ is a multiple of 15, $15N$ is a multiple of 20, $20N$ is a multiple of 25 and $25N$ is a multiple of 30. What is the least possible value of N?

$$10N = 15j,$$

$$15N = 20k,$$

$$20N = 25\ell,$$

$$25N = 30m,$$

$$j, k, \ell, m \in \mathbb{Z}^+$$

Compute the least possible value of N.

$$2N = 3j,$$
$$3N = 4k,$$
$$4N = 5\ell,$$
$$5N = 6m,$$
$$j, k, \ell, m \in \mathbb{Z}^+$$

$$2N = 3j \implies 3 \mid N,$$
$$3N = 4k \implies 2^2 \mid N,$$
$$4N = 5\ell \implies 5 \mid N,$$
$$5N = 6m \implies 2 \mid N \land 3 \mid N,$$
$$j, k, \ell, m \in \mathbb{Z}^+$$

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$$\implies 2^2 \cdot 3 \cdot 5 \mid N$$
$$N \in \mathbb{Z}^+ \implies N = \text{lcm}(\{2^2, 3, 5\}) = \boxed{60}$$

Concepts

- definition of multiple
- reason about restrictions (divisibility, least common multiple)