Contest Problem Set 12213

Sprint Round Problem 25

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Let x be a positive real number such that

$$(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5.$$

What is the value of x^2?

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$(x-1)(x^9+x^8+\cdots+x^2+x)$$

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$(x-1)(x^9+x^8+\cdots+x^2+x)=x^{10}+x^9+\cdots+x^2$$

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$(x-1)(x^9+x^8+\cdots+x^2+x) = x^{10}+x^9+\cdots+x^2 \\ -x^9-\cdots-x^2-x$$

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$(x-1)(x^9+x^8+\cdots+x^2+x)+x=x^{10}+x^9+\cdots+x^2+x$$
$$-x^9-\cdots-x^2-x$$

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$\begin{aligned}(x-1)(x^9+x^8+\cdots+x^2+x)+x &= x^{10}+x^9+\cdots+x^2+x\\ &\quad -x^9-\cdots-x^2-x\\ &= x^{10}\end{aligned}$$

Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$\begin{aligned}(x-1)(x^9+x^8+\cdots+x^2+x)+x &= x^{10}+x^9+\cdots+x^2+x\\ &\quad -x^9-\cdots-x^2-x\\ &= x^{10}\\ &= 2021^5\end{aligned}$$

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Given $(x-1)(x+x^2+\cdots+x^k+\cdots+x^9)+x=2021^5$, find x^2.

$$\begin{aligned}(x-1)(x^9+x^8+\cdots+x^2+x)+x &= x^{10}+x^9+\cdots+x^2+x \\ &\quad -x^9-\cdots-x^2-x \\ &= x^{10} \\ &= (x^2)^5 = 2021^5 \\ \implies x^2 &= \boxed{2021}\end{aligned}$$

Concepts

- distributive property
- properties of exponents
- equating base expressions