

# Contest Problem Set 12128

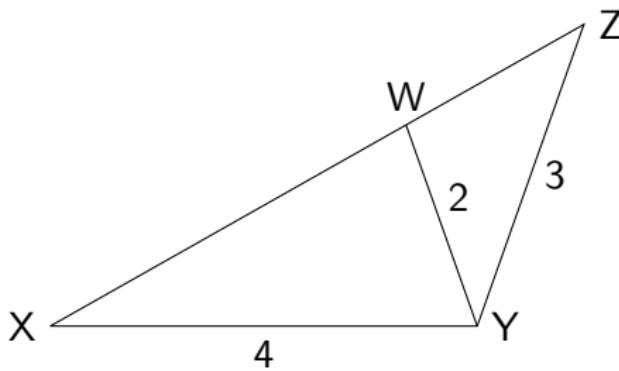
## Sprint Round Problem 29

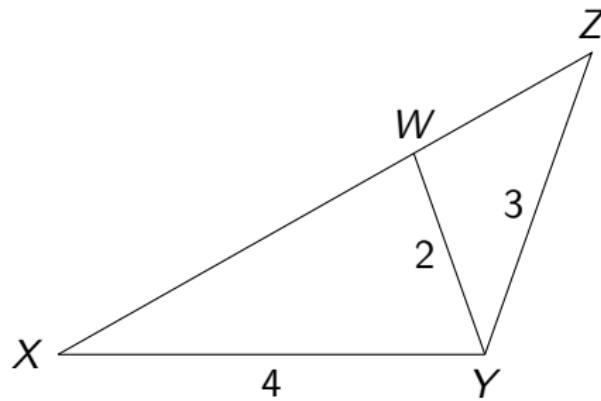
**David Sun**

Math League, LLC

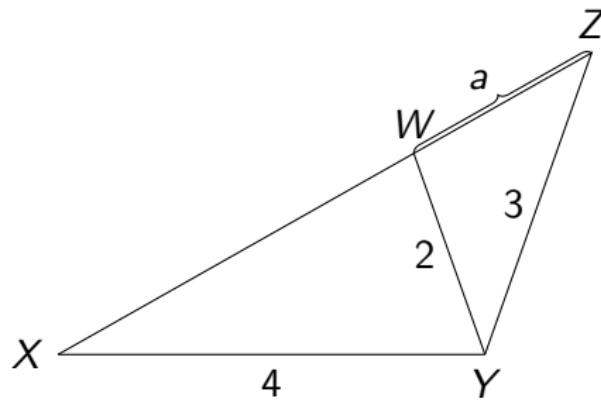


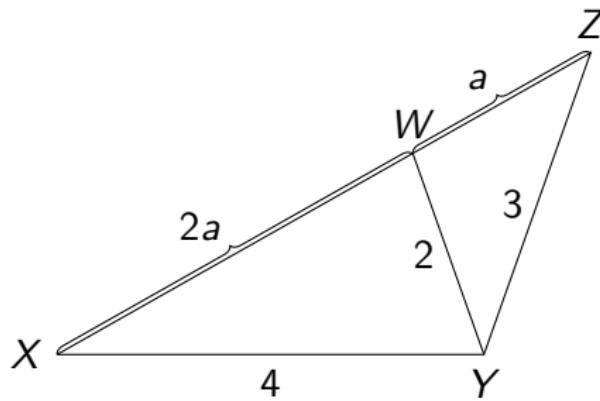
In  $\triangle XYZ$ , side  $XY$  has length 4 and side  $YZ$  has length 3. Point  $W$  lies on side  $XZ$  such that the length of segment  $YW$  is 2, and the length of segment  $XW$  is twice the length of segment  $WZ$ . What is the square of the length of side  $XZ$ ?

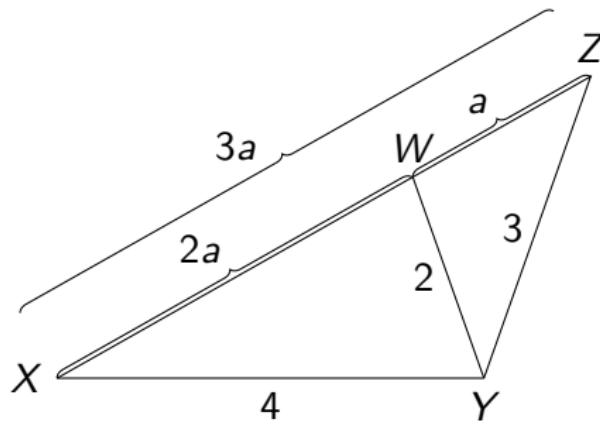




Compute  $XZ^2$ .

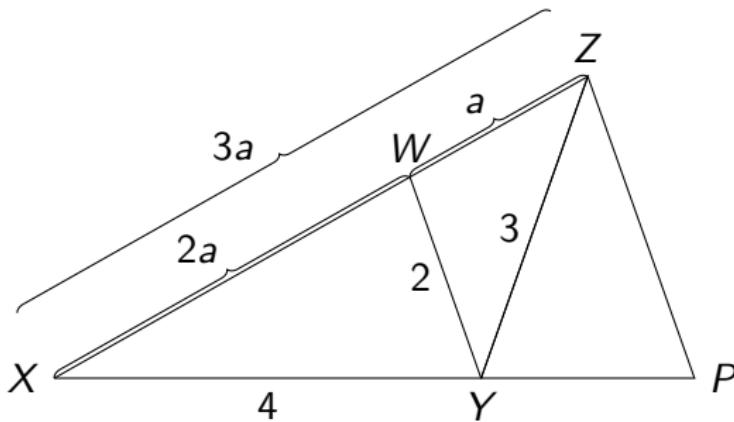






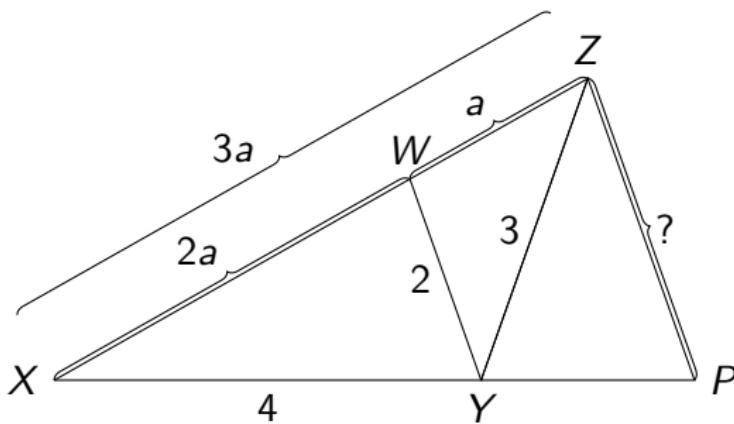
Compute  $XZ^2$ .

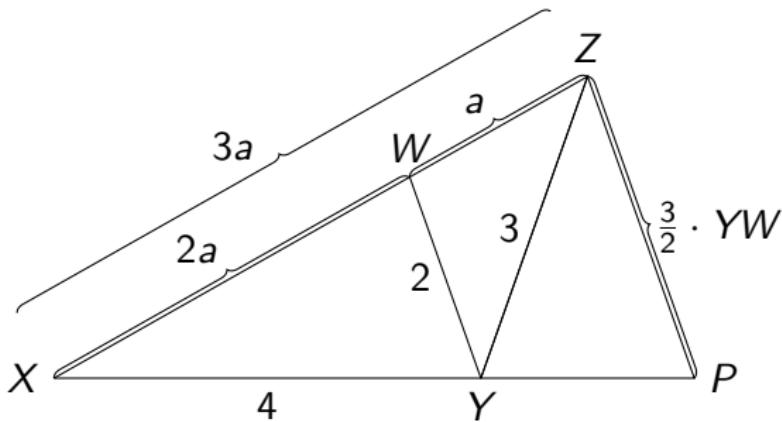
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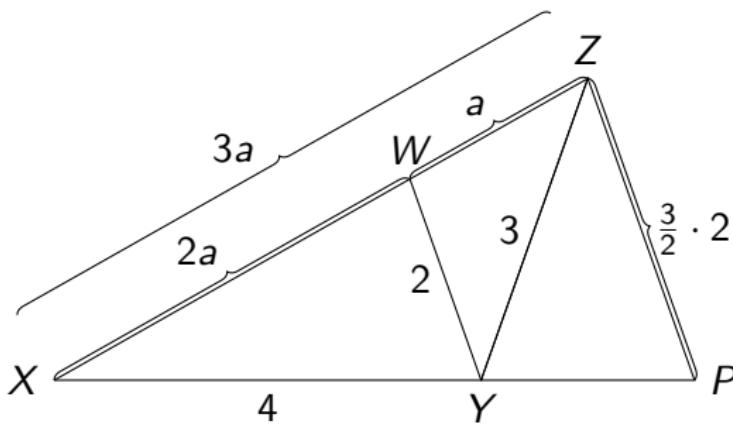


Compute  $XZ^2$ .

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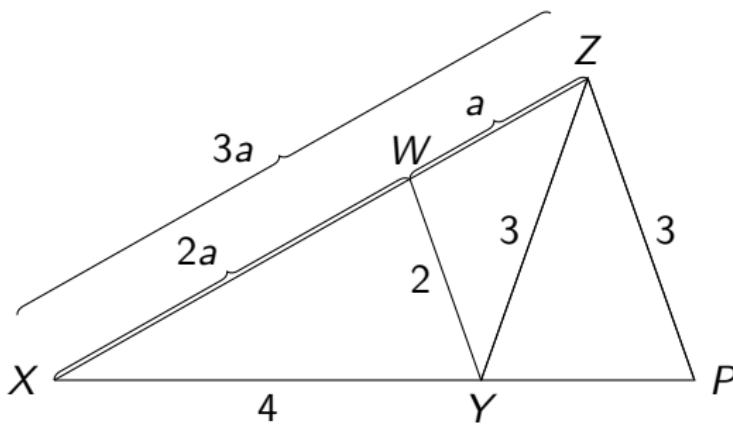


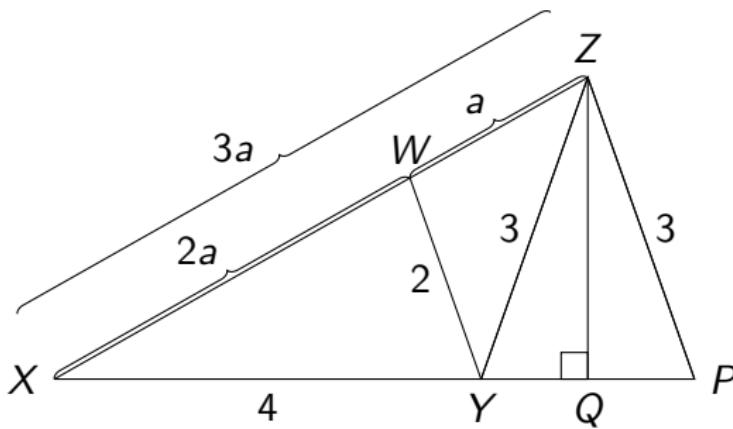
Compute  $XZ^2$ .                    

Compute  $XZ^2$ . oooooooooooooooo●ooooooooooooooo

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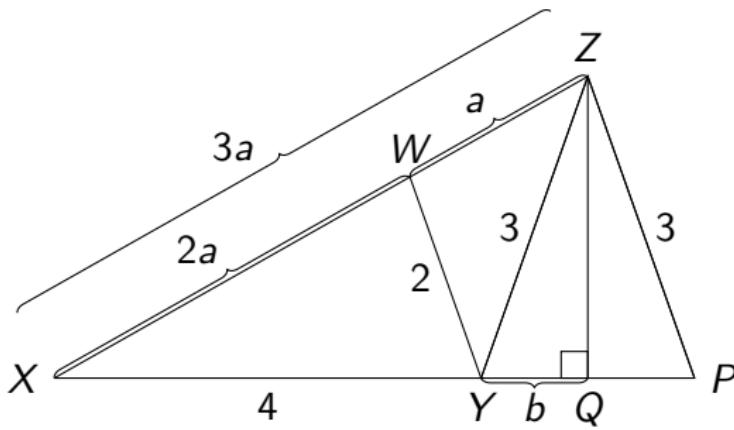




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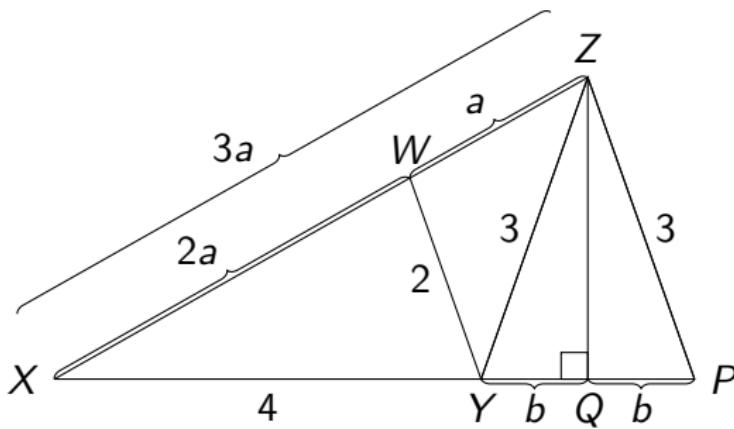
Compute  $XZ^2$ .

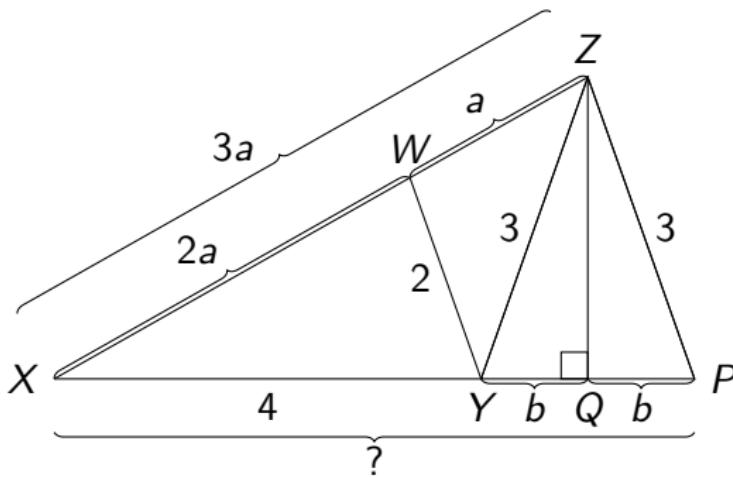


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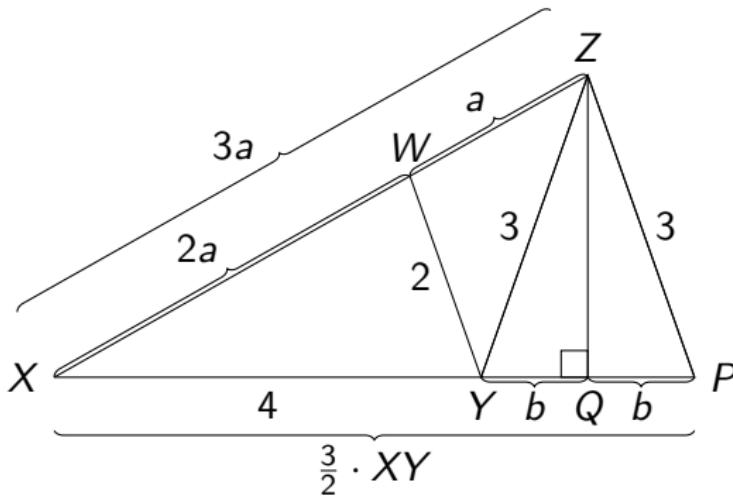
Compute  $XZ^2$ .





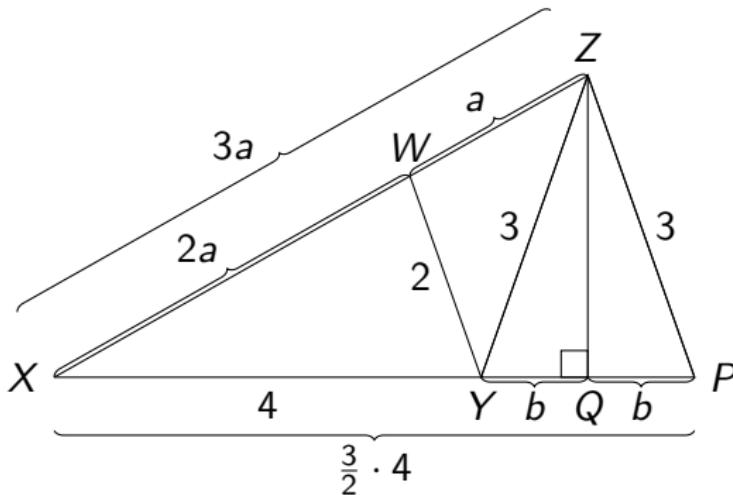
Compute  $XZ^2$ .

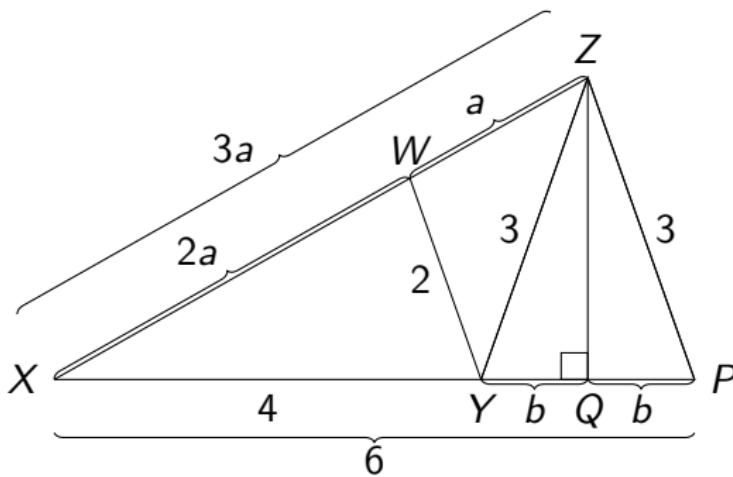
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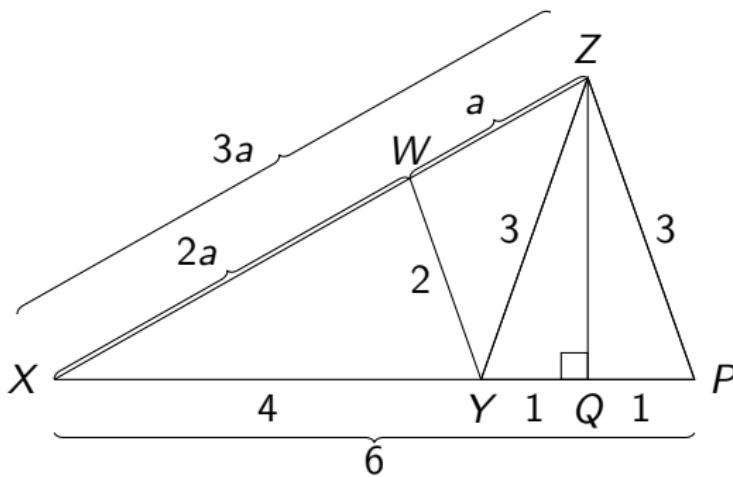


Compute  $XZ^2$ .

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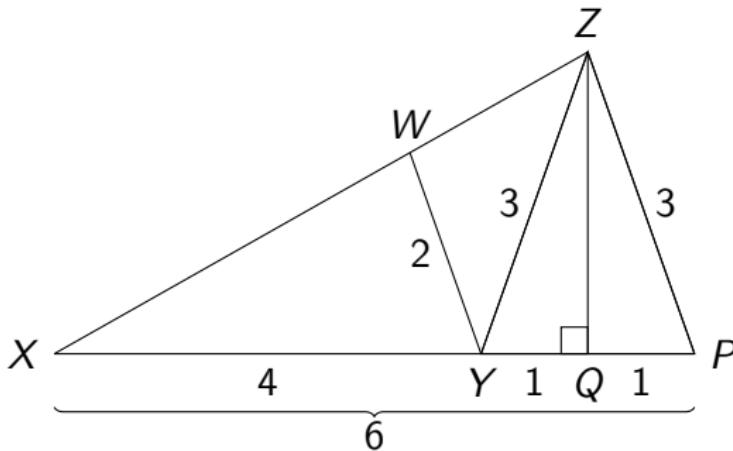


Compute  $XZ^2$ . oooooooooooooooo●ooooooooooooooo



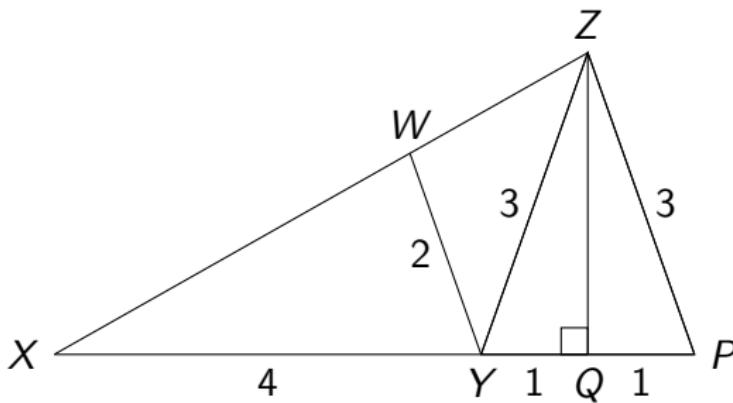
Compute  $XZ^2$ .

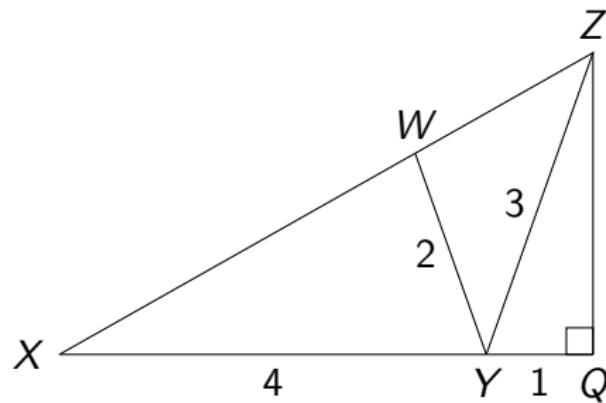
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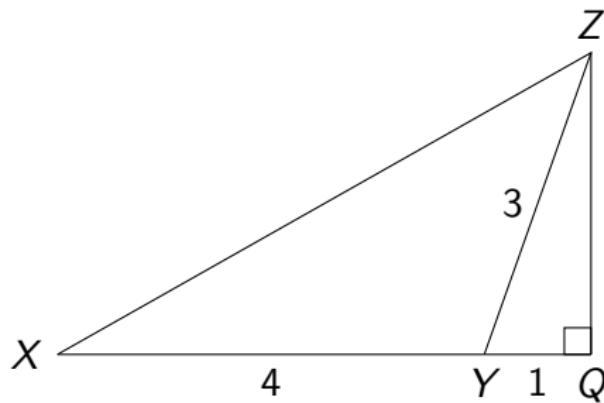


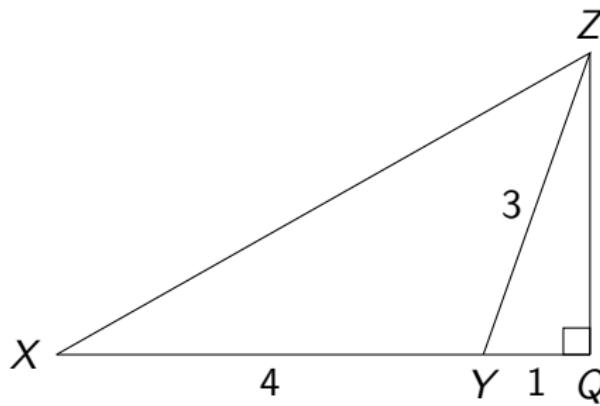
Compute  $XZ^2$ .

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Compute  $XZ^2$ .○○○○○○○○○○○○○○○○●○○○○○○○○○○○○

Compute  $XZ^2$ .oooooooooooooooooooo●oooo

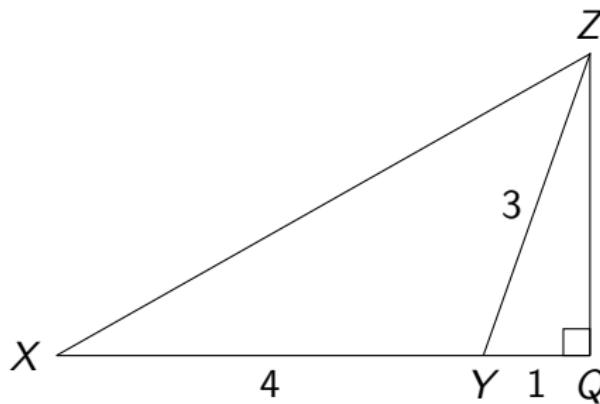
Compute  $XZ^2$ .oooooooooooooooooooo●oooo

By Pythagorean theorem,

$$XZ^2 = XQ^2 + QZ^2$$

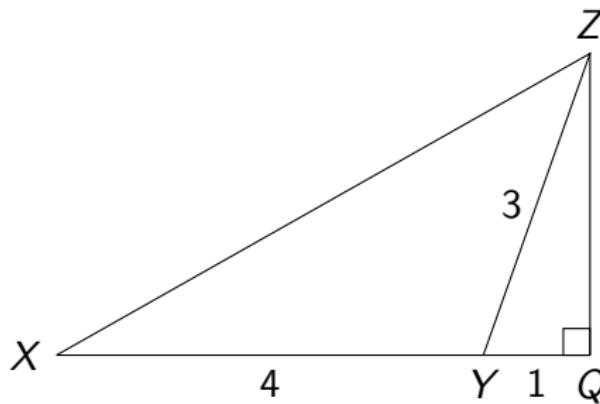
Compute  $XZ^2$ .

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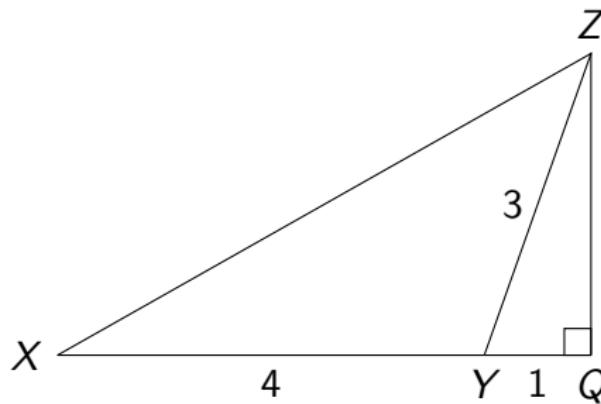


By Pythagorean theorem,

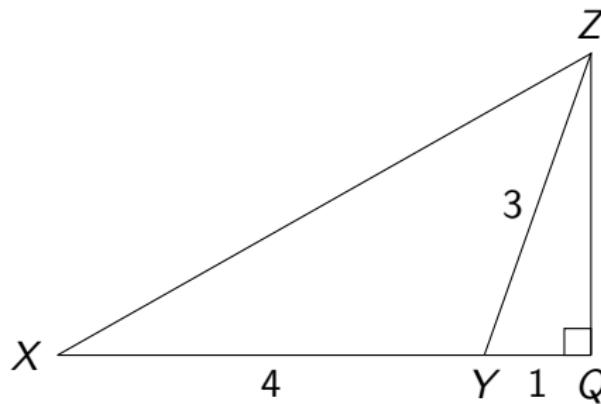
$$XZ^2 = XQ^2 + (YZ^2 - YQ^2)$$

Compute  $XZ^2$ .oooooooooooooooooooo●oooo

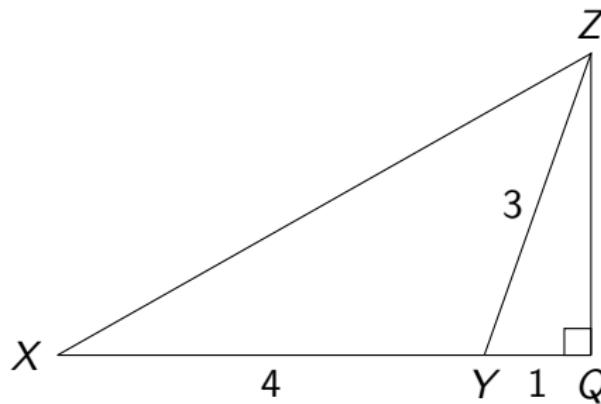
$$XZ^2 = XQ^2 + (YZ^2 - YQ^2)$$

Compute  $XZ^2$ .oooooooooooooooooooo●ooooooo

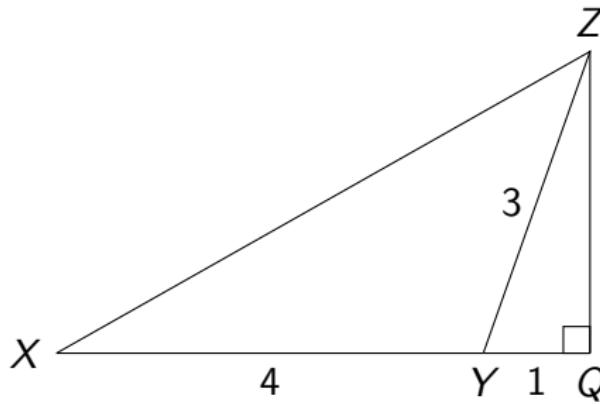
$$XZ^2 = (XY + YQ)^2 + (YZ^2 - YQ^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooooo●ooooooo

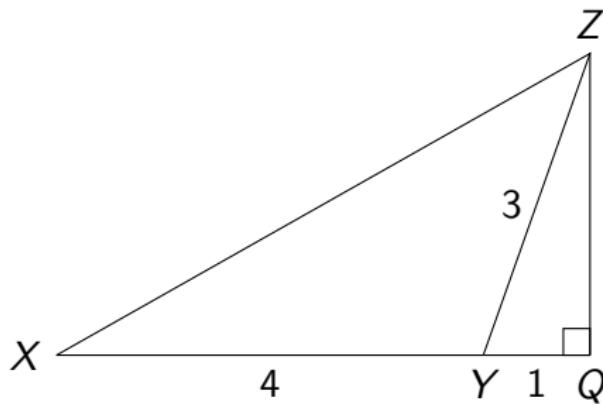
$$XZ^2 = (4 + YQ)^2 + (YZ^2 - YQ^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooo●ooooooo

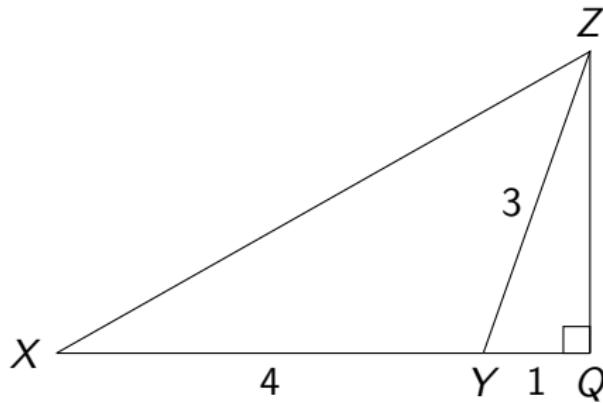
$$XZ^2 = (4 + 1)^2 + (YZ^2 - 1^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooooo●oooooo

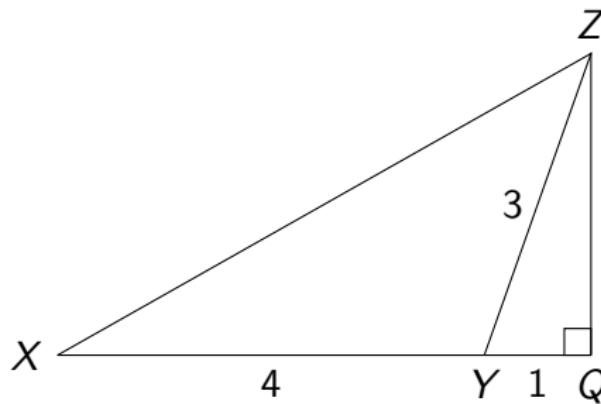
$$XZ^2 = (4 + 1)^2 + (3^2 - 1^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooooo●ooooo

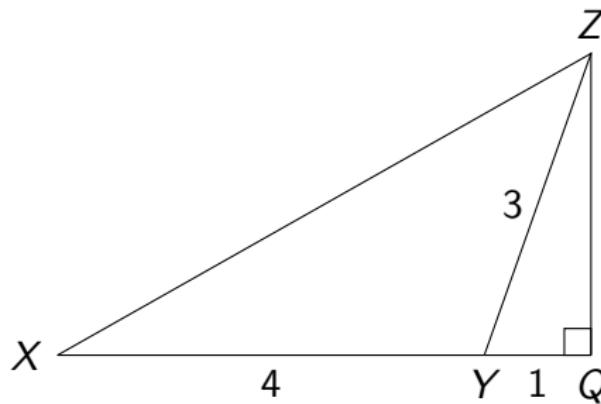
$$XZ^2 = 5^2 + (3^2 - 1^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooooo●oooo

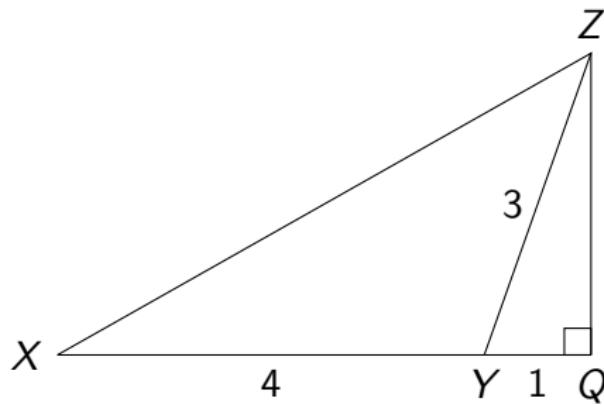
$$XZ^2 = 25 + (3^2 - 1^2)$$

Compute  $XZ^2$ .ooooooooooooooooooooooo●oooo

$$XZ^2 = 25 + (9 - 1^2)$$

Compute  $XZ^2$ .oo●○○

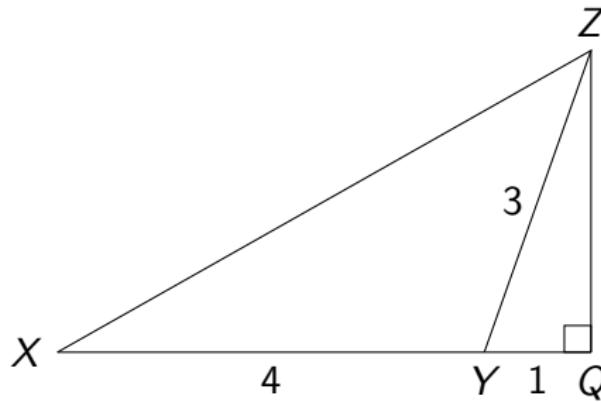
$$XZ^2 = 25 + (9 - 1)$$

Compute  $XZ^2$ .ooo

$$XZ^2 = 25 + 8$$

Compute  $XZ^2$ .

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$$XZ^2 = \boxed{33}$$

Review the concepts.



# Concepts

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- similar triangles

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- properties of isosceles triangles

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- Pythagorean theorem