Contest Problem Set 12114

Team Round Problem 3

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A whole number value is assigned to every letter in the English alphabet. The *score* of a word is the sum of the values of each letter in the word. Suppose the scores of the words *ERRANT*, *RERUN*, and *AUNT* are 67, 49, and 37 respectively, and the value of N is the average of the values of A and T. What is the value that was assigned to the letter N?

A whole number value is assigned to every letter in the English alphabet. The *score* of a word is the sum of the values of each letter in the word. Suppose the scores of the words *ERRANT*, *RERUN*, and *AUNT* are 67, 49, and 37 respectively, and the value of N is the average of the values of A and T. What is the value that was assigned to the letter N?

Objective: Compute the value assigned to the letter N.

Let the value of each letter be denoted by its lowercase.

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$a =$ the value of A

Compute the value assigned to the letter N.

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

Compute the value assigned to the letter N.

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n$$

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n$$

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n = 49,$$

Compute the value assigned to the letter N.

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n = 49,$$

$$a + t + u + n$$

Let the value of each letter be denoted by its lowercase.

$a =$ the value of A

$b =$ the value of B

$\vdots$

$$2 \cdot r + e + a + t + n = 67,$$

$$2 \cdot r + e + u + n = 49,$$

$$a + t + u + n = 37.$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$(2) - (3)$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$

$$(2) - (3) \implies 2 \cdot r + e - (a + t) = 12$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$(1) - (4)$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$(1) - (4) \implies 2 \cdot (a + t) + n = 55$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$

$$2 \cdot r + e + u + n = 49 \quad (2)$$

$$a + t + u + n = 37 \quad (3)$$

$$2 \cdot r + e - (a + t) = 12 \quad (4)$$

$$2 \cdot (a + t) + n = 55 \quad (5)$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t)$$

Compute the value assigned to the letter N.

$$2 \cdot r + e + a + t + n = 67 \tag{1}$$
$$2 \cdot r + e + u + n = 49 \tag{2}$$
$$a + t + u + n = 37 \tag{3}$$
$$2 \cdot r + e - (a + t) = 12 \tag{4}$$
$$2 \cdot (a + t) + n = 55 \tag{5}$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$2 \cdot 2 \cdot n + n = 55.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$4 \cdot n + n = 55.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$(4 + 1) \cdot n = 55.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$5 \cdot n = 55.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$\frac{1}{5} \cdot 5 \cdot n = \frac{1}{5} \cdot 55.$$

$$2 \cdot r + e + a + t + n = 67 \quad (1)$$
$$2 \cdot r + e + u + n = 49 \quad (2)$$
$$a + t + u + n = 37 \quad (3)$$
$$2 \cdot r + e - (a + t) = 12 \quad (4)$$
$$2 \cdot (a + t) + n = 55 \quad (5)$$

Given that the value of N is the average of the values of A and T,

$$n = \frac{1}{2} \cdot (a + t) \implies 2 \cdot n = a + t.$$

Substituting $2 \cdot n$ for $(a + t)$ in (5), we have that

$$n = \boxed{11}.$$

Concepts

Concepts

- elimination

Concepts

- elimination
- substitution