Contest Problem Set 12113

Team Round Problem 3

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Math League, LLC

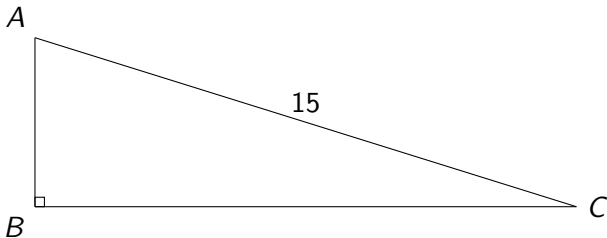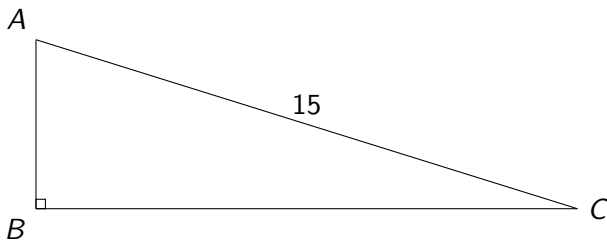In right-angled $\triangle ABC$, the hypotenuse has a length of 15. If the perimeter of $\triangle ABC$ is $15 + \sqrt{353}$, what is the area of $\triangle ABC$?
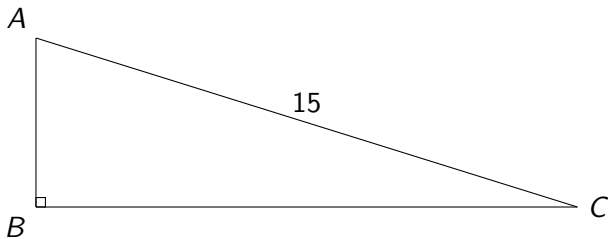
A

15

B C

Perimeter of $\triangle ABC = 15 + \sqrt{353}$

A

15

B C

$$AB + BC + AC = 15 + \sqrt{353}$$

Compute the area of $\triangle ABC$.
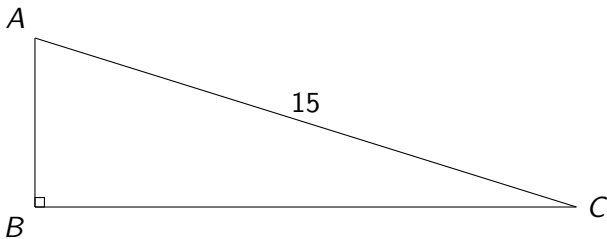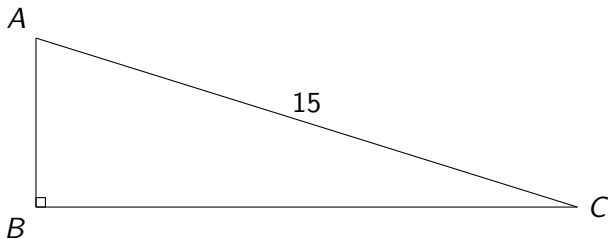
$$AB + BC + 15 = 15 + \sqrt{353}$$

A

15

B

C

$$AB + BC = \sqrt{353}$$

$$AB + BC = \sqrt{353}$$

Area of $\triangle ABC$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

A

15

C

B

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(AB + BC)^2$$

A

15

C

B
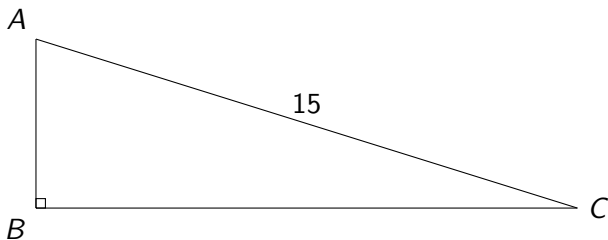
$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(AB + BC)^2 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$
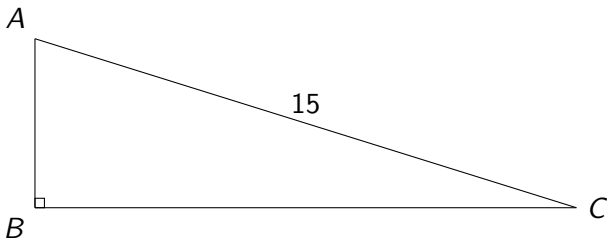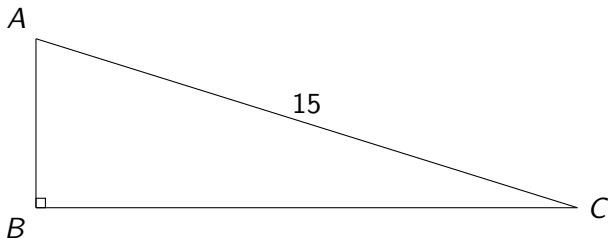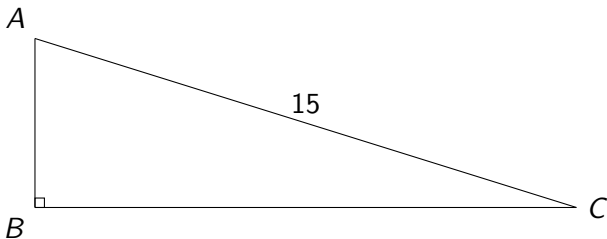
Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$(\sqrt{353})^2 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$
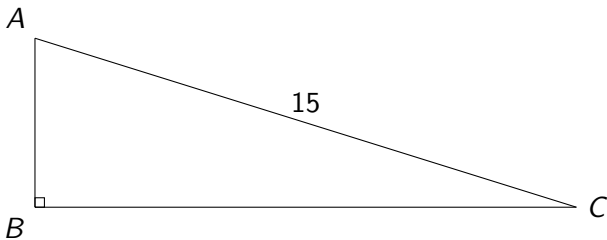
$$353 = AB^2 + 2 \cdot AB \cdot BC + BC^2$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + (2 \cdot AB \cdot BC + BC^2)$$

Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

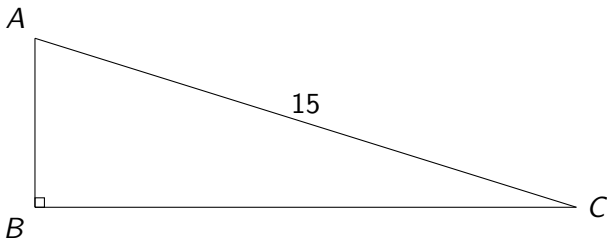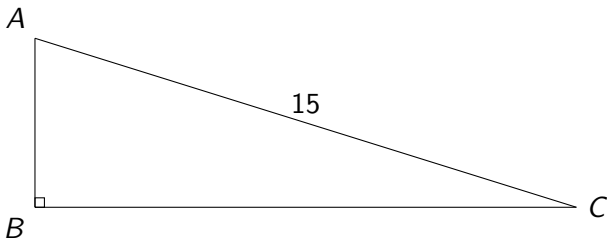$$353 = AB^2 + (BC^2 + 2 \cdot AB \cdot BC)$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = AB^2 + BC^2 + 2 \cdot AB \cdot BC$$

Compute the area of $\triangle ABC$.
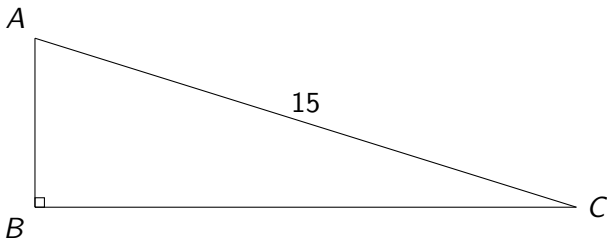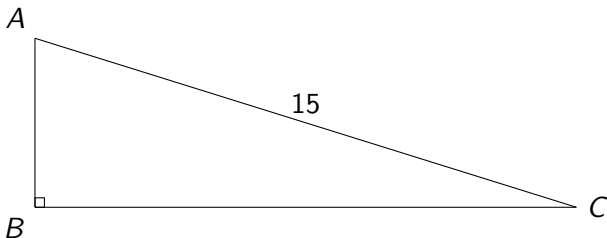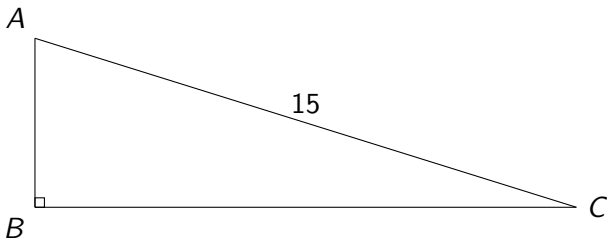

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 = (AB^2 + BC^2) + 2 \cdot AB \cdot BC$$

Compute the area of $\triangle ABC$.

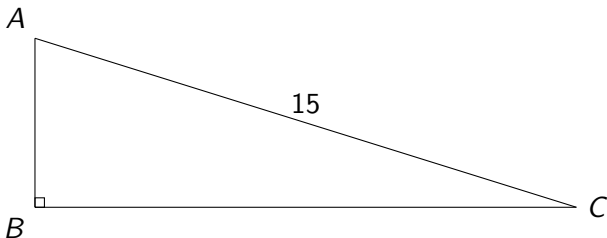$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$353 - (AB^2 + BC^2) = 2 \cdot AB \cdot BC$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = AC^2$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

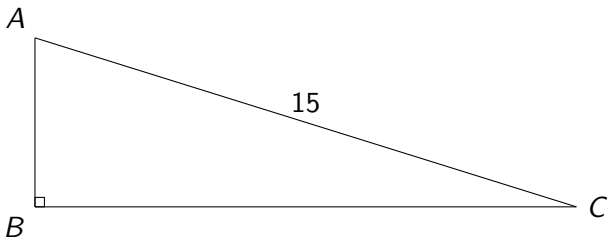$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = 15^2$$
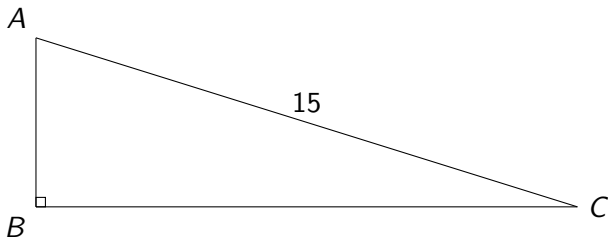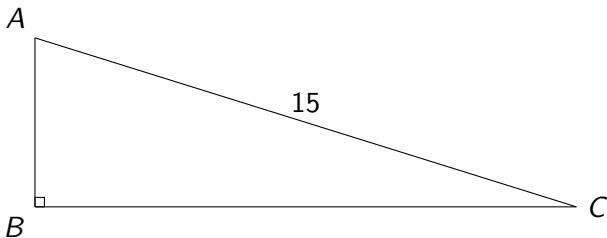
Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

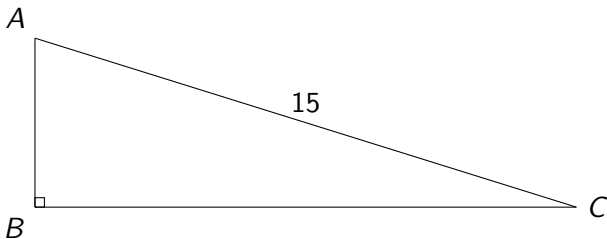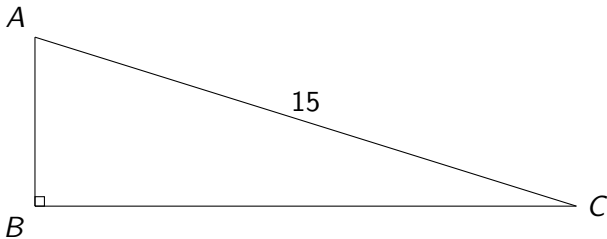$$2 \cdot AB \cdot BC = 353 - (AB^2 + BC^2)$$

$$AB^2 + BC^2 = 225$$

Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$
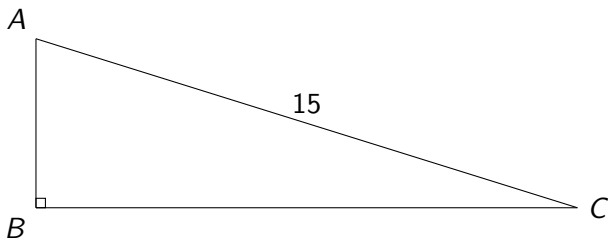
$$2 \cdot AB \cdot BC = 353 - 225$$

$$AB^2 + BC^2 = 225$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$2 \cdot AB \cdot BC = 128$$

$$AB^2 + BC^2 = 225$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

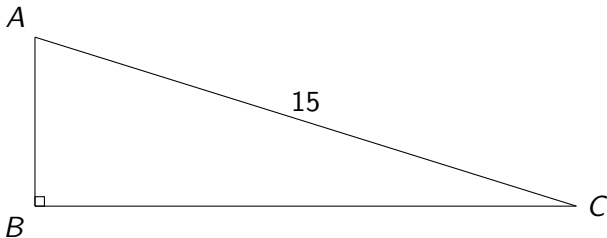$$\frac{1}{2} \cdot 2 \cdot AB \cdot BC = \frac{1}{2} \cdot 128$$
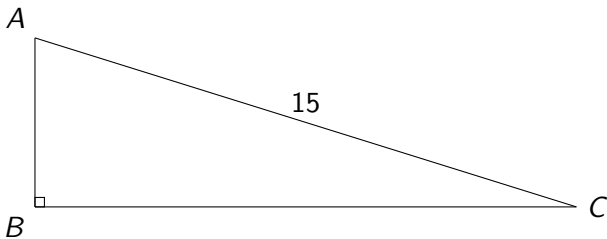
$$AB^2 + BC^2 = 225$$

Compute the area of $\triangle ABC$.

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot AB \cdot BC$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot 64$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

$$AB + BC = \sqrt{353}$$

$$\text{Area of } \triangle ABC = \boxed{32}$$

$$AB \cdot BC = 64$$

$$AB^2 + BC^2 = 225$$

Review the concepts.

Concepts

Concepts

- perimeter and area of a triangle

Concepts

- perimeter and area of a triangle
- algebraic manipulation

Concepts

- perimeter and area of a triangle
- algebraic manipulation
- Pythagorean theorem